The University of Nottingham

DEPARTMENT OF MECHANICAL, MATERIALS AND MANUFACTURING ENGINEERING

A LEVEL 2 MODULE, AUTUMN SEMESTER 2013-2014

MECHANICS OF SOLIDS 2

Time allowed TWO Hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer FOUR questions

Only silent, self contained calculators with a Single-Line Display or Dual-Line Display are permitted in the examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

Several of our exam papers require students to answer a specific number of questions from those available, e.g. answer 4 out of 6 questions. If students answer more than the required number of questions only the required number will be marked, so for the example given, only the first 4 solutions in the exam script will be marked.

If during the exam students attempt additional questions they should clearly indicated on their script which solutions they want to be marked - simply putting a line through solutions that should be disregarded is recommended.

ADDITIONAL MATERIAL: Graph Paper

INFORMATION FOR INVIGILATORS:

Question papers should be collected in at the end of the exam – do not allow candidates to take copies from the exam room.

Turn over

1. An aero engine drive shaft of outer diameter 61.5mm with a wall thickness of 3mm is subjected to an axial force, F, torque, T and bending moment, M at various points during service as shown in Figure Q1.

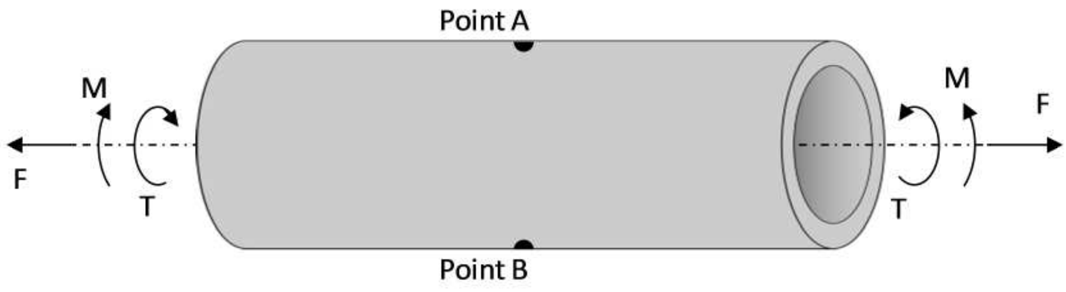

Figure Q1

During cruise, the bending moment is negligible and can be ignored and the shaft is subjected to the following loads:

F = 60kN
T = 3.5kNm

(a) Calculate the stresses on the outer surface of the shaft due to each of the applied loads. [6 marks]

(b) Sketch the stress state of a plane stress element on the outer surface of the shaft at point A and after sketching Mohr's circle, determine the values of the principal stresses, σ_1 and σ_2, and the maximum shear stress, τ_{max} at that point. [8 marks]

(c) At takeoff the shaft is subjected to a significant bending moment in addition to the axial and torque loading, given the following loads:

F = 100kN
T = 5kNm
M = 1.5kNm

Using sketches of the stress state and Mohr's circle, determine the values of the principal stresses, σ_1 and σ_2, and the maximum shear stress, τ_{max} at both point A and point B on the outer surface of the shaft. [11 marks]

2. A steel beam which is simply supported at both ends, of length 5m, is subjected to two point loads, P_1 and P_2, and a uniformly distributed load, q, as shown in Figure Q2.

P_1 = 4kN P_2 = 7kN q = 8kN/m E = 210GPa I = 2×10^{-5}m^4

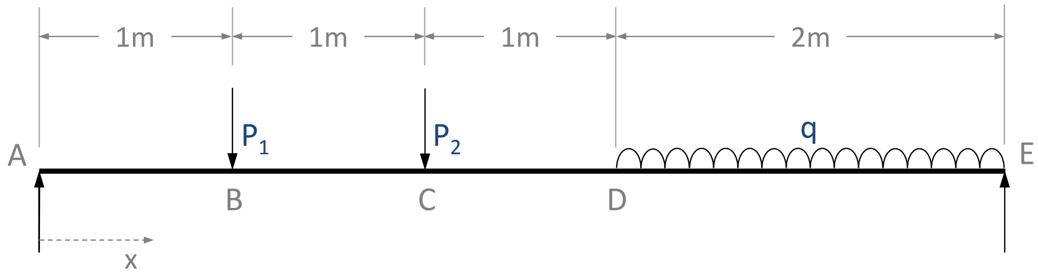

Figure Q2

(a) Determine the reaction forces, R_A and R_E, at the beam supports. [5 marks]

(b) Section the beam in the span between points D and E and draw a free body diagram for the beam to the left of this section. [3 marks]

(c) Calculate the slope and deflection at the position of the point load P_2 (i.e. at x = 2m). [12 marks]

(d) Calculate the deflection of the beam 1m from the right end of the beam (i.e. at x = 4m). [5 marks]

3. The cross-section of a straight I - section beam is shown in Figure Q3. It is loaded in pure bending, about the Y-Y axis until the whole of each flange has yielded and the whole of the web remains elastic.

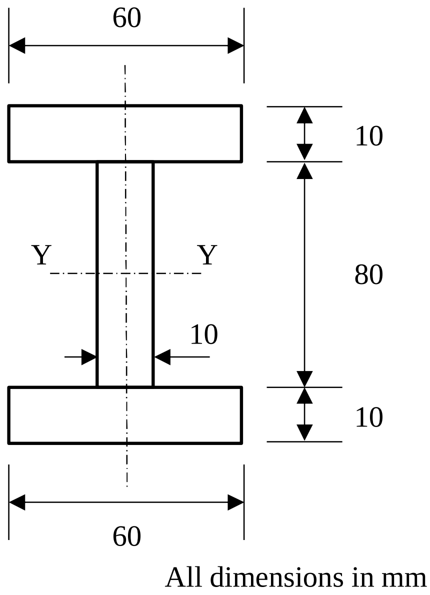

Figure Q3

(a) Determine the bending moment required and the radius of curvature of the beam when this moment is applied. [13 marks]

(b) Show the residual stress state in the beam and determine the residual radius of curvature of the beam when this moment is removed. [12 marks]

The material can be assumed to be elastic-perfectly-plastic with a yield stress of 250MPa and a Young's modulus of 200 GPa.

4. (a) Define by means of an equation the stress intensity factor and give examples of the effects of finite boundaries on stress intensity factors. [3 marks]

(b) Describe with the aid of diagrams the relationship between crack growth rate and stress intensity factor. [6 marks]

(c) Show with the aid of diagrams the effects of mean stress on fatigue crack growth. [6 marks]

(d) Investigation of a thin-walled pressure vessel of diameter 1000mm and thickness 10mm reveals a longitudinal surface crack of 8mm in length with a geometry correction factor, Y, of 1.12. Given that the vessel is made from steel with a fracture toughness of 75 $MN/m^{3/2}$ and that a safety factor of 2 is required, determine the safe working pressure of the vessel assuming linear elastic material. [10 marks]

5. (a) Give an equation for the direct thermal strain seen due to a temperature change. [3 marks]

(b) Show the Generalized Hooke's law equations incorporating the thermal strain term. [5 marks]

(c) A rectangular beam, width b and depth d, has a temperature variation given by:

$$T = T_o \left\{ 1 - \frac{4y^2}{d^2} \right\}$$

where y is the distance from the neutral axis, and T_0 is a temperature constant.

There is no restraint or applied loading (ie P = M = O).

Determine the bending stress distribution in the beam. [17 marks]

Turn over

6. The Tresca or τ_{max} criterion states that the material will yield if:

$$\sigma_1 - \sigma_3 \geq \sigma_y \qquad (\sigma_1 > \sigma_2 > \sigma_3)$$

The von Mises yield criterion states that the material will yield if:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \geq 2\sigma_y{}^2$$

(a) Show the yield boundaries for Tresca and von Mises, for a two-dimensional stress state, on the σ_1-σ_2 plane (σ_3=0). [3 marks]

(b) Represent the yield surfaces for Tresca and von Mises for a three-dimensional stress state. [3 marks]

(c) On (b) show the decomposition of the stress into hydrostatic and deviatoric components. [3 marks]

(d) A solid circular bar with a diameter of 50mm is subjected to an axial load, P and a torque, T. The yield stress of the material is 250MPa. If the value of the axial load, P, is 195kN, considering both the Tresca and von Mises yield criteria, determine the maximum value of torque, T, in each case, which will just cause yielding to occur in the bar. [16 marks]

End